

**Assignment 4:**  
Acoustic Waves

STUDENT #: \_\_\_\_\_

Released: Friday Feb 3

Due: Fri Feb 10 6:00PM

NAME: \_\_\_\_\_

- 1.A)** An earthquake on the ocean floor in the Gulf of Alaska produces a *tsunami* (sometimes incorrectly called a "tidal wave") that reaches Hilo, Hawaii, 4 450 km away, in a time interval of 9 h 30 min. Tsunamis have enormous wavelengths (100 to 200 km), and the propagation speed for these waves is  $v \approx \sqrt{g\bar{d}}$ , where  $\bar{d}$  is the average depth of the water. From the information given, find the average wave speed and the average ocean depth between Alaska and Hawaii. (This method was used in 1856 to estimate the average depth of the Pacific Ocean long before soundings were made to give a direct determination.)

$$v = \frac{4\,450 \text{ km}}{9.50 \text{ h}} = 468 \text{ km/h} = \boxed{130 \text{ m/s}} \quad \bar{d} = \frac{v^2}{g} = \frac{(130 \text{ m/s})^2}{(9.80 \text{ m/s}^2)} = \boxed{1730 \text{ m}}$$

- 1B)** The ocean floor is underlain by a layer of basalt that constitutes the crust, or uppermost layer, of the Earth in that region. Below this crust is found denser periodotite rock, which forms the Earth's mantle. The boundary between these two layers is called the Mohorovicic discontinuity ("Moho" for short). If an explosive charge is set off at the surface of the basalt, it generates a seismic wave that is reflected back out at the Moho. If the speed of this wave in basalt is 6.50 km/s and the two-way travel time is 1.85 s, what is the thickness of this oceanic crust?

$$v = \frac{2d}{t} : d = \frac{vt}{2} = \frac{1}{2}(6.50 \times 10^3 \text{ m/s})(1.85 \text{ s}) = \boxed{6.01 \text{ km}}$$

- 2** A simple pendulum consists of a ball of mass  $M$  hanging from a uniform string of mass  $m$  and length  $L$ , with  $m \ll M$ . If the period of oscillations for the pendulum is  $T$ , determine the speed of a transverse wave in the string when the pendulum hangs at rest.

The period of the pendulum is  $T = 2\pi\sqrt{\frac{L}{g}}$ . Let  $F$  represent the tension in the string (to avoid confusion with the period) when the pendulum is vertical and stationary. The speed of waves in the string is then:

$$v = \sqrt{\frac{F}{\mu}} = \sqrt{\frac{Mg}{\frac{m}{L}}} = \sqrt{\frac{MgL}{m}}. \text{ Since it might be difficult to measure } L \text{ precisely, we eliminate}$$

$$\sqrt{L} = \frac{T\sqrt{g}}{2\pi} \quad \text{so } v = \sqrt{\frac{Mg}{m}} \frac{T\sqrt{g}}{2\pi} = \boxed{\frac{Tg}{2\pi} \sqrt{\frac{M}{m}}}.$$

- 3A** A loudspeaker is placed between two observers who are 110 m apart, along the line connecting them. If one observer records a sound level of 60.0 dB, and the other records a sound level of 80.0 dB, how far is the speaker from each observer?

- 3B** If a single person in the stands of a stadium shouts, the intensity level at the centre of the field is 50dB. What is the intensity level when  $2 \times 10^4$  spectators are shouting from the same distance?

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- 4 Show that the difference between decibel levels  $\beta_1$  and  $\beta_2$  of a sound is related to the ratio of the distances  $r_1$  and  $r_2$  from the sound source by

$$\beta_2 - \beta_1 = 20 \log \left( \frac{r_1}{r_2} \right)$$

We begin with  $\beta_2 = 10 \log \left( \frac{I_2}{I_0} \right)$ , and  $\beta_1 = 10 \log \left( \frac{I_1}{I_0} \right)$ , so  $\beta_2 - \beta_1 = 10 \log \left( \frac{I_2}{I_1} \right)$ .

Also,  $I_2 = \frac{\rho}{4\pi r_2^2}$ , and  $I_1 = \frac{\rho}{4\pi r_1^2}$ , giving  $\frac{I_2}{I_1} = \left( \frac{r_1}{r_2} \right)^2$ . Then,  $\beta_2 - \beta_1 = 10 \log \left( \frac{r_1}{r_2} \right)^2 = \boxed{20 \log \left( \frac{r_1}{r_2} \right)}$ .

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- A) What is the length of a closed organ pipe with a fundamental frequency of 24Hz?  
What is the length of open organ pipe that has a fundamental frequency of 500Hz?
- B) The speed of sound varies with temperature: at 20°C it is 344m/s, while at 5°C it is 335m/s.  
Consider an open pipe of length 30cm. By how much does its fundamental frequency change when the temperature drops from 20°C to 5°C? Assume that the length is unchanged.

A  $\left( f = \frac{v}{\lambda} \text{ and } L = \frac{\lambda}{4} \right) \Rightarrow L = \frac{1}{4} \frac{v}{f} = \frac{1}{4} \frac{335}{24} = 3.49\text{m}$

B  $\left( f = \frac{v}{\lambda} \text{ and } L = \frac{\lambda}{4} \right) \Rightarrow L = \frac{1}{4} \frac{v}{f} = \frac{1}{4} \frac{335}{500} = 0.1675\text{m}$

C  $f = \frac{1}{4} \frac{v}{L} = \frac{344}{4 \times 0.3} = 286.67\text{Hz}$   $f = \frac{1}{4} \frac{v}{L} = \frac{335}{4 \times 0.3} = 279.17\text{Hz}$

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A police car whose siren has a natural frequency of 1000Hz approaches a large wall at 90km/h.  
A stationary observer detects the direct and reflected waves.

- A) What is the beat frequency if the observer is between the car and the wall,?
- B) what is the beat frequency if the observer is behind the car?
- C) A bat, moving at 5.00 m/s, is chasing a flying insect. If the bat emits a 40.0-kHz chirp and receives back an echo at 40.4 kHz, at what speed is the insect moving toward or away from the bat? (Take the speed of sound in air to be  $v = 340$  m/s.)

6AB analyzed during lecture:

6C solved in DGD. ANS:  $v = 3.31\text{m/s}$  away from the bat. NOTE if original assumption is made that the insect flies towards the bat than, the resulting velocity will be negative ( $-3.31\text{m/s}$ ) which indicates opposite direction to the one assumed originally.

3A

Let  $r_1$  and  $r_2$  be the distance from the speaker to the observer that hears 60.0 dB and 80.0 dB, respectively. Use the result of problem 28,

$$b_2 - b_1 = 20 \log \left( \frac{r_1}{r_2} \right), \text{ to obtain } 80.0 - 60.0 = 20 \log \left( \frac{r_1}{r_2} \right).$$

Thus,  $\log \left( \frac{r_1}{r_2} \right) = 1$ , so  $r_1 = 10.0 r_2$ . Also:  $r_1 + r_2 = 110 \text{ m}$ , so

$$10.0 r_2 + r_2 = 110 \text{ m giving } \boxed{r_2 = 10.0 \text{ m}}, \text{ and } \boxed{r_1 = 100 \text{ m}}.$$

3B

For single shouter  $\beta = 10 \log \frac{I}{I_0}$  and so  $50 = 10 \log \frac{I}{I_0} \Rightarrow 10^5 = \frac{I}{I_0} \Rightarrow I = 10^{-7} \frac{W}{m^2}$

For 20000 spectators:

$$I = 20000 \times 10^{-7} \frac{W}{m^2} = 2 \times 10^{-3} \frac{W}{m^2} \text{ and the sound level is}$$

$$\beta = 10 \log \frac{2 \times 10^{-3} \frac{W}{m^2}}{I_0} = 10 \log(2 \times 10^9) = 117 \text{ dB}$$